PHILOSOPHICAL TRANSACTIONS.

I. *On the Determination of* VERDET'S *Constant in Absolute Units. By* J. E. H. GORDON, *B.A., Gonville and Caius College, Cambridge.—1st and 2nd Memoirs*. Communicated by* J. CLERK MAXWELL.

Received June 5,—Read June 15, 1876.

[PLATE 1.]

[NOTE.—The whole of this work has been done under Professor CLERK MAXWELL'S superintendence; he suggested the method and nearly all the details, and any merit which the investigation may have belongs to him. He is, however, in no way responsible for any errors there may be in the numerical results.]

CONTENTS.

* In a former memoir which Prof. MAXWELL did me the honour of communicating to the Royal Society, and which was read June 17th, 1875, the object of this research was explained and a result given.

The method employed, however, was faulty in several respects, the two most important of which were:—

1st. The rotations produced were exceedingly small, so that a slight error in the determination of the plane of polarization introduced a large error into the results.

2nd. A tangent-galvanometer was used to determine the strength of the current.

In the experiments which are the subject of the present Memoir I have, at Prof. MAXWELL'S suggestion, used

INTRODUCTORY.

In the year 1845 Faraday discovered that if plane polarized light passes through certain media, and these media be acted on by a sufficiently powerful magnetic force, the plane of polarization is rotated.

About the year 1853 M. Verdet commenced a long and exhaustive examination of the subject, and his first result was published in Ann. de Chimie et de Phys. 3 sér. tom. xli.

In Maxwell's 'Electricity,' Art. 808, vol. ii. p. 400, M. Verdet's results are summarized as follows:—

The angle through which the plane of polarization is turned is proportional

(1) To the distance which the ray travels within the medium;

(2) To the intensity of the resolved part of the magnetic force in the direction of the ray.

(3) The amount of rotation depends on the nature of the medium.

Prof. Clerk Maxwell then goes on to say

"These three statements are included in the more general one that the angular rotation is numerically equal to the amount by which the magnetic potential increases from the point at which the ray enters the medium to that at which it leaves it *multiplied by a coefficient* which for diamagnetic media is generally positive."

The object of the present research is to determine the coefficient for a particular medium and for light of a particular wave-length.

"Verdet's constant" is this coefficient for a particular medium and ray, or, in other words, it is the angular rotation produced on that ray and in that medium by a difference of magnetic potential equal to unity.

In order that the measurements may be expressed in absolute units, it is necessary to modify M. Verdet's mode of proceeding in several respects.

In particular an electromagnet with an iron core is unsuitable for this investigation, for both the amount and the distribution of the magnetic force between the poles depend on the properties of the iron core, and cannot be deduced from the strength of the current in the helix. Faraday's heavy glass and other media having the highest power of rotating the plane of polarization were also unsuitable to be used as standard media on account of the difficulty of procuring specimens exactly alike. The following method was therefore adopted:—

The magnetic force was produced by means of an electric current in a helix without an iron core, and bisulphide of carbon enclosed in a tube with glass ends placed within the helix was chosen as the medium. The constants of the helix were measured by the methods described in pages 4–14.

The strength of the current in it was deduced from the deflection of a small magnet

bisulphide of carbon instead of water, which was used in the earlier experiments, thus increasing the rotation, and have made the helix act as its own galvanometer by suspending a magnetized needle in its neighbourhood.

In consequence of these improvements in the methods, I have, through Prof. Maxwell, requested the Council of the Royal Society to permit me to withdraw my first Memoir, and have at the same time embodied in the present paper those portions of the first which are necessary for a due understanding of the second.

suspended near to it and outside it, and the rotation was measured by means of a divided circle.

The methods by which the light was polarized and analyzed are described later. The investigation then resolved itself into three parts:—

(1) The determination of the constants of the helix.

(2) The determination of the ratio which the rotation per unit of length bore to the tangent of the deflection of the suspended needle.

(3) The determination of the horizontal component of the earth's magnetism at the time and place of observation.

The greater number of the experiments were made in my laboratory at Pixholme, near Dorking, but the determination of the number of windings and part of the determination of the area of the helix were made in the Cavendish laboratory in Cambridge.

DESCRIPTION OF INSTRUMENTS EMPLOYED.

The helix was about 26·34 centims. long, about 5 centims. internal diameter, and 12·5 centims. external, and contained about 35 lbs. of No. 20 cotton-covered copper wire. The insulation-resistance was exceedingly high, sufficiently so to enable me to neglect leakage. The resistance of the helix was 1·01 B.A. unit.

The *small dynamometer* consisted of an ebonite ring, about 12 centims. diameter, on which were wound six turns of wire. It was made for me by Messrs. ELLIOTT, and the mean diameter of the coils accurately gauged.

The *large dynamometer* was the instrument belonging to the British Association. It is described and figured in MAXWELL'S 'Electricity,' vol. ii. fig. 54, p. 330. (The suspended coil there shown was not used.)

The polarizing apparatus consisted of a Nicol's prism, rather more than 1 centim. diameter, fixed into a collimating tube taken from a spectroscope. The slit was removed, and the Nicol substituted in its place.

The analyzing apparatus.—This, together with its circle and the Nicol, is the property of the Cavendish laboratory, and was kindly lent to me by Professor MAXWELL. The analyzer was a Jellett prism, described in the next paragraph. It was mounted in a tube, together with a lens and some diaphragms, which gave very good definition (see Plate 1).

*The Jellett prism** consisted of a piece of Iceland spar, of which the ends were cut normal to the sides. It was then divided by a plane, making an angle of about 1° with the plane containing the two long diagonals. One half was reversed, and the two cemented together again.

The prism was mounted in a brass tube, and a diaphragm, with a hole of some 3 or 4 millims. diameter in it, was placed across one end. On looking in from the other end of the prism, the hole was seen by the ordinary rays in the form of a circle divided by a

* This prism is described by Prof. JELLETT in the 'Transactions of the Royal Irish Academy,' vol. xxv. Prof. JELLETT also had the kindness to advise the author as to the form of his instrument best suited to this investigation.

line across it, the light of the two semicircles being polarized in planes making with each other an angle of about 2°.

The image of the hole formed by the extraordinary rays consisted of two semicircles, one to the right and the other to the left of this circle.

A second diaphragm hid the latter. Now when light that has passed through a Nicol is examined with the prism, and the latter turned so that the light of one half of the circle is extinguished, the other half is slightly illuminated. If the Jellett be turned through rather less than two degrees, the second half of the circle will become dark, and the first will be slightly illuminated.

It is obvious that there is a position between these two where the illumination over the whole circle is uniform, and this position can be observed with considerable accuracy.

With the arrangements that were used in this work the probable error was about 1′.

The divided circle was made for this investigation by Mr. BROWNING, and is about 14 centims. diameter. The circle turns against two fixed verniers. It is divided on brass to 1°, and the verniers read to $\frac{1}{20}^{\circ}$, *i. e.* to 3′. By estimation 1′ can be read with perfect accuracy. It is moved by a screw of long pitch, working in a thread cut round its edge. This gives a sufficiently quick motion to enable changes in the illumination of any part of the field to be noted, and is yet capable of very delicate adjustment.

The galvanometer consisted of a minute suspended magnet and mirror, weighing about 1 grain, acted on by the helix (outside of which it was placed), and observed by a telescope and scale.

The tube for containing the bisulphide was of brass, with glass ends, and projected some 20 centims. from each end of the helix.

The experimenting table was built of massive brickwork, with a carefully levelled top of fine cement. It was about 1·4 metre square. A continuation of it supported the suspended magnet, and a separate brick pier the telescope by which the magnet was observed (see Plate 1).

The apparatus for measuring horizontal distances consisted of a number of conical plumb-bobs, which hung by silk threads from laths laid across a horizontal deal frame the same size as the table, and fixed about 1 metre above it. Distances projected on the table by these were taken by compasses, and measured on a millim. scale engraved on a massive brass plate by ELLIOTT (for another purpose).

The base of the circle, the telescopes, &c. were all placed on pillars of brickwork built on the tables, and bedded either in cement or plaster of Paris. The result was a steadiness such as I have never before obtained.

THE EXPERIMENTS.

Determination of the Number of Windings.

We must first determine the difference of magnetic potential at the two ends of the helix when a unit current passes through it. It is a quantity which we call N, and is a function of the number of windings and their arrangement; for if we know the magnetic intensity at each point of the axis of the helix due to a unit current, viz. what force is exerted by a unit current in the helix on a unit magnetic pole at that point, then we know how much work would have to be done to move this unit pole from one end of the helix to the other when a unit current was passing in it.

But if a, b be the ends and u_x the force at any point, then the above amount of work would be equal to

$$\int_a^b u_x dx = V_b - V_a, \quad \ldots \ldots \ldots \quad (1)$$

where V is the potential at any point.

But the dimensions of magnetic potential are, in the electromagnetic system,

$$[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}];$$

and these are also the dimensions of the strength of an electric current.

∴ N, which is the ratio of the former of these things to the latter, *is a number*.

The value of N for the helix was determined by comparison with the great dynamometer of the British Association, which is deposited in the Cavendish Laboratory.

The intensities of the magnetic action were compared at 7 equidistant points in the axis of the helix, and the total force was obtained by integrating by WEDDLE's formula (BOOLE's 'Finite Differences,' p. 47), viz.

$$\int_0^{6h} u_x\, dx = \tfrac{3}{10}h\{u_0 + u_2 + u_4 + u_6 + 5(u_1 + u_5) + 6u_3\}^*, \quad \ldots \ldots \quad (2)$$

where h is the distance between any two of the points, and u_x is the magnetic intensity at any point in terms of that of the dynamometer.

The mechanical arrangements were as follows:—the dynamometer, which consists of two coils about 50 centimetres in diameter, at a distance of about 25 centimetres one from the other, was placed so that the plane of the coils was accurately vertical and in the magnetic meridian.

The axial line of the coils was then carefully found and marked by means of plumb-lines and cross-threads fixed to the table.

A strong T-shaped board supported on three levelling-screws was placed so that the part corresponding to the stem of the T passed through the coils in a horizontal plane parallel to their axis; on this the helix was laid and its axis brought into exact coincidence with that of the coils.

A boxwood cylinder about 20 centims. long was turned to fit nicely into the helix; a long thick brass wire terminating in a handle was fixed into one end, and a brass pin about 5 centims. long was fixed near one edge of the other.

* Objections have been taken to the use of this formula, which gives, it is observed, much more weight to u_1 and u_5 than to u_2 and u_4, and does not furnish an approximation of a legitimate analytical character. While fully acknowledging the force of these objections, I have not thought it worth while to make an alteration which would involve repeating most of the arithmetic in the paper, for this reason: the experiments, being made with resistance-coils, are susceptible of such close accuracy that the errors of any particular determination, even when multiplied by 5, cannot perceptibly affect the value of N deduced.

To the end of the latter a magnet and mirror weighing only about a grain was hung by a silk fibre, so that when the pin was at the highest point the magnet hung in the axis of the cylinder. The helix being placed coaxial with the coils was pushed endways so that the centre point of the coils was just outside one end of it.

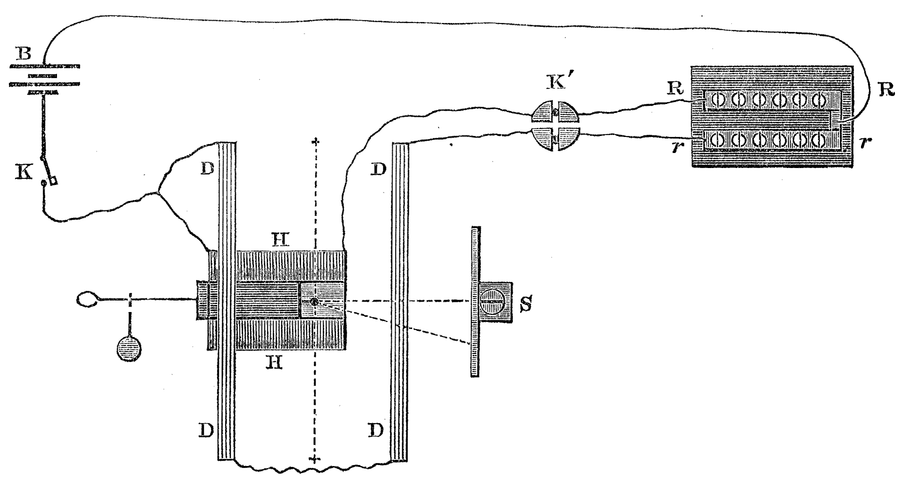
The cylinder was then inserted and adjusted, by means of cross-wires, so that the mirror hung exactly at the centre point, and a mark was then made on the handle corresponding to a mark on the stand of the instrument, by reference to which the magnet could always be brought to the same position.

Thus while the helix was slid along the axis, the magnet inside it could always be placed at the centre point of the dynamometer. The distance between the inside ends of the helix was divided into six equal parts by means of seven pins stuck as sights into a slip of wood fixed along the top edge. Thus by sliding the helix along till any one of the sights was between the vertical threads placed midway between the coils, the force at that point due to the helix could be compared with that due to the dynamometer by means of the needle at the centre.

The comparison was made by sending currents of different intensities through the helix and coils in opposite directions, and varying them till there was no action on the needle.

This was effected by dividing a current and passing one portion through the coils and the other through the helix, and interposing different resistances in each branch.

The annexed diagram shows the connections.



B is the battery, R and r the resistance-coils.
D, the dynamometer-coils. } And let these letters also represent
H, the helix. } their resistances.
● the magnet and mirror, whose angular position is observed by
S, the scale and lamp.
K K′ are contact-keys.

Now when the actions on the needle are equal, the powers of the coils are inversely as the currents in them, that is, directly as the resistances $r+D$ and $R+H$.

The observations being repeated at each of the seven points, we have the intensity of the magnetic action of the helix at each of those points in terms of that of the dynamometer.

Thus if P be the power of the dynamometer with unit current, and u_x the power of the helix with the same current at a point x, then the total force of the helix (that is, the difference of magnetic potentials at its ends) is

$$N=\int_0^{6h} u_x\,dx=\frac{3h}{10}P\left\{\frac{H+R_0}{D+r_0}+\frac{H+R_{2h}}{D+r_{2h}}+\frac{H+R_{4h}}{D+r_{4h}}+\frac{H+R_{6h}}{D+r_{6h}}\right.$$
$$\left.+5\left(\frac{H+R_h}{D+r_h}+\frac{H+R_{5h}}{D+r_{5h}}\right)+6\,\frac{H+R_{3h}}{D+r_{3h}}\right\}, \quad \ldots \ldots \quad (3)$$

where h is $\frac{1}{6}$ the length of the helix.

In the helix used the following results were obtained:—

$$h=4{\cdot}39 \text{ centims.}, \quad u_x=P\frac{R+H}{r+D}, \quad \begin{matrix} H=\ 1{\cdot}01, & \text{take}=1{\cdot}00, \\ D=28{\cdot}15 & ,, \quad 28{\cdot}1. \end{matrix}$$

	$r=100.$	$r=200.$	$r=1000.$
u_0	$P\frac{348+H}{100+D}=2{\cdot}724\,P$	$P\frac{629+H}{200+D}=2{\cdot}762\,P$	$P\frac{2797+H}{1000+D}=2{\cdot}722\,P$
u_1	$P\frac{636+H}{100+D}=4{\cdot}973\,P$	$P\frac{1134+H}{200+D}=4{\cdot}976\,P$	$P\frac{5100+H}{1000+D}=4{\cdot}962\,P$
u_2	$P\frac{725+H}{100+D}=5{\cdot}667\,P$	$P\frac{1290+H}{200+D}=5{\cdot}659\,P$	$P\frac{5840+H}{1000+D}=5{\cdot}681\,P$
u_3	$P\frac{739+H}{100+D}=5{\cdot}777\,P$	$P\frac{1315+H}{200+D}=5{\cdot}725\,P$	$P\frac{5960+H}{1000+D}=5{\cdot}798\,P$
u_4	$P\frac{713+H}{100+D}=5{\cdot}574\,P$	$P\frac{1270+H}{200+D}=5{\cdot}572\,P$	$P\frac{5750+H}{1000+D}=5{\cdot}592\,P$
u_5	$P\frac{615+H}{100+D}=4{\cdot}808\,P$	$P\frac{1097+H}{200+D}=4{\cdot}814\,P$	$P\frac{4969+H}{1000+D}=4{\cdot}639\,P$
u_6	$P\frac{348+H}{100+D}=2{\cdot}724\,P$	$P\frac{620+H}{200+D}=2{\cdot}722\,P$	$P\frac{2799+H}{1000+D}=2{\cdot}723\,P$

Hence we have for the values of u_x, by taking the means of the above,—

r.	u_0.	u_1.	u_2.	u_3.	u_4.	u_5.	u_6.
$r=$ 100	2·724 P	4·973 P	5·667 P	5·777 P	5·574 P	4·808 P	2·724 P
$r=$ 200	2·762 P	4·976 P	5·659 P	5·725 P	5·572 P	4·814 P	2·722 P
$r=$1000	2·722 P	4·962 P	5·681 P	5·798 P	5·592 P	4·639 P	2·723 P
Mean ...	2·736 P	4·970 P	5·669 P	5·766 P	5·579 P	4·754 P	2·723 P

Hence

$$N=\int_0^{6h} u_x\,dx=\tfrac{3}{10}\,4\cdot39 \text{ centims.}\{16\cdot707+48\cdot620+34\cdot596\}P$$

$$=131\cdot732[LP]P_D\text{*}.$$

Now $$P_D=81\cdot1620,$$

$$\therefore\ 131\cdot732\,P_D=10751\cdot96.$$

Now in order that this may be a number it is necessary that [P] should equal $[L^{-1}]$; and if we consider the equation of moments we shall see that this is so. For consider a small magnet (length $2l$) at the centre of the dynamometer, we have, when a current passes,

$$H\sin\delta\, l\,[H\,.\,L]=PC\cos\delta\, l\,[PC\,.\,L],$$

i. e.

$$[H]=[P\,.\,C],$$

or

$$[P]=\left[\frac{H}{C}\right]=\frac{[L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]}{[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]}=[L^{-1}]. \qquad (4)$$

Now the value of N for any helix with unit current taken with respect to the whole length of its axis produced to an infinite length in both directions is, by Art. 676 of Professor CLERK MAXWELL'S 'Electricity,' $4\pi n$, where n is the number of windings.

When the length l is finite compared with the radius a, the value of N for that part of the axis which is included between the ends is

$$N=4\pi n\frac{\sqrt{l^2+a^2}-a}{l} \qquad (5)$$

(see CLERK MAXWELL'S 'Electricity,' Art. 676).

Now if we calculate $n=\frac{N}{4\pi}\frac{l}{\sqrt{l^2+a^2}-a}$, taking a the mean radius, this will give the number of windings in the helix.

Now as $a=4\cdot84$ centims., and $N=10752$, we have

$$n=\frac{10752}{4\pi}\cdot\frac{26\cdot34}{\{26\cdot34^2+4\cdot84^2\}^{\frac{1}{2}}-4\cdot84};$$

$$\therefore\ \log n=\log 10752+\log 26\cdot34-\log 4\pi-\log\overline{(l^2+a^2)^{\frac{1}{2}}-a}$$

{last term = log 21·92}

$$=4\cdot0314893+1\cdot4206158-1\cdot0991971-1\cdot3408405$$

$$=3\cdot0120680,$$

$$\therefore\ n=1028\cdot15,$$

∴ there are 1028 windings on the helix.

* Professor CLERK MAXWELL'S method of representing the units by quantities in [ ] has been used throughout the paper. Thus a length L is represented by L[L], where L is a number and [L] the unit of length.

Verification.

Now in the length of the helix there are 91 windings on the outside layer, and the ratio of the length to the difference of the internal and external radii is $\frac{26\cdot34}{3\cdot47}=7\cdot5$ about.

∴ assuming that the number of layers per centim. of radius is the same as the number of windings per centim. of length (it would really be a little greater, as they fit into each other, ∘∘∘∘), we have

$$n=\left\{91\times\frac{91}{7\cdot5}\right\}=1092,$$

which is sufficiently near the calculated result, viz. $n=1028$, to show that no large mistake has been made, as, for instance, writing down a log with a wrong index.

Closer agreement could hardly be expected, as the helix was made for a different purpose, and no particular pains were taken to wind it uniformly. It is also probable that the instrument-maker took more pains to wind the wires of the outside layer, which could be seen, close together than those of the inside ones, which were hidden.

Determination of Areas.

1*st method.*—To calculate the strength of a current in a helix from the deflection of a needle placed outside it, it is necessary to know $\Sigma(A)$ where A_k is the area of any winding.

To find $\Sigma(A)$ the helix was so placed that a vertical plane through its centre, and normal to its axis, contained the magnet and mirror when only acted on by the earth's magnetism. In this plane, between the helix and magnet, and so arranged that it could be slid along in it, was placed the small dynamometer. The centres of magnet, dynamometer, and helix were in the same horizontal line.

The same current was sent in opposite directions through both helix and small dynamometer*, and the latter moved till there was no action in the magnet. The distances of the centres from the magnet were then found† to be :—

Centre of helix to magnet	82·80 centims.
„ dynamometer to magnet . . .	18·10 „

2*nd method.*—From these data a value of the area was obtained. On looking through these experiments, however, Professor MAXWELL was of opinion that the dynamometer used was not large enough to ensure accuracy, so he had the kindness to compare the helix with the great B.A. dynamometer at Cambridge. In his experiments the helix and dynamometer were placed concentric, and different currents were sent opposite ways

* The circular base of the dynamometer was cemented to a square piece of glass, which slid on a little wooden stand, furnished with a guide-bar lying in the magnetic meridian.

† See paragraphs on measurement of horizontal distances and determination of magnetic meridian, pp. 4 & 16.

through both, till there was no action on a small magnet suspended at a point O in a vertical plane perpendicular to and bisecting the common axis of the dynamometer and helix. Professor MAXWELL found that when from O to centre was so great that the diameters of the coils could be neglected in comparison, the currents producing equilibrium had the ratio

$$1:11\cdot 23.$$

From these data another value of the area was calculated. In both cases the following reasoning was used:—

Calculation of $\Sigma(A)$, the sum of the areas of the windings of the Helix.

We proceed as follows:—

(1) We first obtain an expression for the force exerted at a point by one winding, at a certain distance and of a certain area, carrying a unit current.

(2) We then, by calculation, find what the action of a certain standard coil of known area would be with a unit current.

(3) We then, by experiment, find what that action is with a certain arbitrary current whose ratio to a second current is known.

(4) We then, by experiment, find what the action of the helix is with this second current, adjusting the ratios till the actions are equal.

(5) This eliminates the variations of the current, and enables us, by means of expression (1), to compare the areas acting in (3) and (4); one of these being known, we can obtain the other.

To find an expression for the force exercised by a circular current in a given direction at a point, we must first find the potential at that point, and then differentiate along the given direction.

For a winding carrying a current of strength i we may substitute a magnetic shell bounded by the winding and of strength i. But by MAXWELL's 'Electricity' (Art. 409), the potential at any point due to a magnetic shell is

$$V=\Phi\omega, \qquad (6)$$

where Φ is the strength of the shell and ω the solid angle subtended by it at the point.

We have therefore to find an expression for ω.

Now in Arts. 670 and 694 we find

$$\omega=\frac{1}{c}\frac{d}{dr}\cdot(Pr); \qquad (7)$$

for when Φ=unity, $V=\omega$, where radius of winding (which in this experimental case becomes a great circle)$=c$. r is the distance from the centre of the circle to the point where the potential is to be found, and P the potential of a stratum of matter of surface density unity spread over a hemisphere bounded by the circle. Expanding this, we obtain in Art. 695 the expression for the potential of a point outside the circle,

$$\omega' = 2\pi \sin^2\alpha \left\{ \frac{1}{2}\frac{c^2}{r^2} Q'_1(\alpha)Q_1(\theta) + \&c. + \frac{1}{i+1} \cdot \frac{c^{i+1}}{r^{i+1}} \cdot Q'_i(\alpha)Q_i(\theta) \right\}, \quad . \quad . \quad (8)$$

where α and θ are the angles in Art. 694.

Let xy be the plane of the coil; then the force perpendicular to it is obtained by differentiating the potential with regard to z, and

$$z = r \cos\theta = r\mu.$$

Then for the force we get, changing the independent variables from r, μ, to x, y, z,

$$\frac{d\omega'}{dz} = \frac{d\omega'}{dr} \cdot \frac{dr}{dz} + \frac{d\omega'}{d\mu} \cdot \frac{d\mu}{dz}; \quad \ldots \quad (9)$$

and remembering that

$$r = \sqrt{x^2+y^2+z^2}, \qquad \frac{dr}{dz} = \frac{z}{r} = \mu,$$

$$\mu = \frac{z}{\sqrt{x^2+y^2+z^2}}, \text{ and } \frac{d\mu}{dz} = \frac{r - \frac{z^2}{r}}{r^2} = \frac{1}{r}(1-\mu^2),$$

$$\frac{d\omega'}{dz} = \mu \frac{d\omega'}{dr} + (1-\mu^2)\frac{1}{r} \cdot \frac{d\omega'}{d\mu}. \quad \ldots \quad (10)$$

But when, as in this case, the point where the potential is required is in the plane of the coil, we have α and θ each $=\frac{\pi}{2}$, which gives $\mu=0$, and reduces the expression (10) to

$$\text{Force perpendicular to plane of coil} = \frac{d\omega'}{dz} = \frac{1}{r} \cdot \frac{d\omega}{d\mu}. \quad \ldots \quad (11)$$

Now by Art. 694, p. 301,

$$\frac{d}{d\mu} \cdot Q_i(\theta) = Q_i'(\theta). \quad \ldots \quad (12)$$

The values of this are given in Art. 698, and in them we must put $\mu=0$.

Hence the force for one winding is

$$\frac{d\omega'}{dz} = \frac{2\pi}{r}\left\{\frac{1}{2}\frac{c^2}{r^2}\left[Q_1'\left(\frac{\pi}{2}\right)\right]^2 + \frac{1}{3} \cdot \frac{c^3}{r^3}\left[Q_2'\left(\frac{\pi}{2}\right)\right]^2 + \&c.\right\}$$

$$= \frac{2\pi}{r}\left\{\frac{1}{2} \cdot \frac{c^2}{r^2} \cdot (1)^2 + \frac{1}{4}\frac{c^4}{r^4} \cdot \left(\frac{3}{2}\right)^2 + \frac{1}{6}\frac{c^6}{r^6} \cdot \left(\frac{15}{8}\right)^2 + \&c.\right\}$$

$$= \frac{\pi}{r}\left(\frac{c^2}{r^2} + \frac{1}{2}\frac{c^4}{r^4}\left(\frac{3}{2}\right)^3 + \frac{1}{3} \cdot \frac{c^6}{r^6}\left(\frac{15}{8}\right)^2\right). \quad \ldots \quad (13)$$

This is the force exercised by one winding of radius c on a unit magnetic pole in its plane distant r from its centre, when the winding carries a unit current.

The area of the winding is

$$A=\pi c^2.$$

The force exercised when the strength of the current is C is of course $C\frac{d\omega'}{dz}$. Then we may write

$$F_0=\frac{\Sigma(A)}{r^3}\left(1+\tfrac{9}{16}\frac{c^2}{r^2}+\tfrac{75}{128}\cdot\frac{c^4}{r^4}+\ldots\right), \quad \ldots \ldots \quad (14)$$

where F_0 is the force on a unit pole due to a unit current.

But the small dynamometer consists of windings all of the same diameter, and approximately all in one plane, and therefore this expression is so nearly true that it may be used for it. When thus used it will be distinguished by dashes on the letters.

The second and third terms are required because in the experiments the small dynamometer is placed very close to the suspended magnet.

The effect of the helix may be got by a much simpler method (see MAXWELL, Art. 676). Let $2l$ be the length of the helix, c its mean radius, then the outside effect at the distance of (say) a metre or more is that of a disc of (+) magnetism at one end and of (−) magnetism at the other.

For unit current the moment of the magnet is $\Sigma(A)$; hence its strength is $\frac{\Sigma(A)}{2l}$, which is the quantity of magnetism at each end.

Then the value of the potential will be $\frac{\Sigma A}{2l}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$, and the value of the force at O will be $F_0=\frac{\Sigma(A)}{2l}\cdot\frac{2}{r^2}\cdot\frac{l}{r}=\frac{\Sigma A}{r^3}$ per unit current.

Thus the force $-F'$ exerted by the dynamometer is

$$-F'=-\frac{C'm\Sigma(A)'}{r_l^3}\left(1+\tfrac{9}{16}\cdot\frac{c_l^2}{r_l^2}+\tfrac{75}{128}\frac{c_l^4}{r_l^4}\right), \quad \ldots \ldots \quad (15)$$

while F, that exerted by the helix, is

$$F=\frac{C\Sigma(A)m}{r^3} \quad \ldots \ldots \quad (16)$$

But in my experiments $C=-C'$; and when we have no action on the magnet at O, we have

$$F=-F'.$$

Adding the two expressions we have

$$0=\frac{\Sigma(A)}{r^3}-\frac{\Sigma(A)'}{r_l^3}\left(1+\tfrac{9}{16}\cdot\frac{c_l^2}{r_l^2}+\tfrac{75}{128}\frac{c_l^4}{r_l^4}\right) \quad \ldots \ldots \quad (17)$$

Now let r, r' be the values of r for the helix and small dynamometer respectively, we have, when the force on the suspended magnet is zero,

$$r^3=(\overline{82\cdot8}^2+b^2)^{\frac{3}{2}}, \text{ where } 2b=26\cdot34 \text{ centims.,}$$

and

$$\therefore r^3=\log^{-1} 5\cdot7703411.$$

Also noting that $2b_{\prime}=1\cdot04$,

$$r_{\prime}^3=(\overline{18\cdot10}^2+\overline{0\cdot52}^2)^{\frac{3}{2}}=\log^{-1} 3\cdot7735723.$$

Now $\Sigma(A)'=6\pi c_{\prime}^2$, where $c_{\prime}$ is the mean radius of the coils.

Now the diameters of the coil are

Internal	4·815 inches*,
External	4·950 ,,

which give a mean radius

$$c_{\prime}=6\cdot201 \text{ centims.};$$

whence $\Sigma(A)'=\log^{-1} 2\cdot8602118$.

From (17) we have, then,

$$\Sigma(A)=\frac{\Sigma(A)'r^3\left(1+\frac{9}{16}\frac{c_{\prime}^2}{r_{\prime}^2}+\frac{75}{128}\frac{c_{\prime}^4}{r_{\prime}^4}\right)}{r_{\prime}^3} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (18)$$

Calculating this from the data just given, we have as the result of the small dynamometer experiments

$$\text{Log } \Sigma(A)=4\cdot8880699$$

and

$$\Sigma(A)=77280\cdot5.$$

From these same experimental data Prof. MAXWELL made a calculation, from which he found

$$\Sigma(A)=77554\cdot0$$

Prof. MAXWELL'S *Experiments with the large Dynamometer.*

In these experiments the helix and dynamometer were placed exactly concentric. A magnet and mirror was suspended rather more than a metre distant from them, first in front and then behind, so as to correct any error in centering, and varying currents were sent opposite ways till there was no deflection.

At this distance the helix and dynamometer could each be considered to be replaced by their equivalent magnetic discs, and the difference between r and r_1 may be neglected in comparison with those quantities themselves.

The formula for $\Sigma(A)$ then becomes

$$\Sigma(A)=\frac{C'}{C}\,\Sigma(A)'.$$

Where there was no action on the suspended magnet, Prof. MAXWELL found

$$\frac{C}{C'}=11\cdot23$$

Now the area $\Sigma(A)'$ of the great dynamometer is

870200 sq. centims.

* These measurements were made by Messrs. ELLIOTT.

This gives

$$\Sigma(A)=77488{\cdot}8 \text{ sq. centims.}$$

Thus we obtain the following values for the area of the helix:—

By Prof. MAXWELL'S experiments with the great B.A. dynamometer—

$$\Sigma(A)=77488{\cdot}8 \text{ sq. centims.}$$

By GORDON'S experiments with small dynamometer (GORDON'S calculation)—

$$\Sigma(A)=77280{\cdot}5 \text{ sq. centims.}$$

By MAXWELL'S calculation of GORDON'S experiments—

$$\Sigma(A)=77554 \text{ sq. centims.}$$

Mean of the two calculations of GORDON'S experiments—

$$\Sigma(A)=77417{\cdot}2 \text{ sq. centims.}$$

Finally,

$$\Sigma(A)=\begin{cases}\text{MAXWELL} \quad . \quad . \quad . \quad . \quad 77488{\cdot}8 \\ \text{GORDON} \quad . \quad . \quad . \quad . \quad . \quad 77417{\cdot}2\end{cases}\Big\} \text{ Mean } 77453{\cdot}0 \text{ sq. centims.}$$

Calculation of the strength C *of a current in the Helix in terms of the deflection* δ *of the suspended magnet.*

Let $2l$ be the length of the suspended magnet, H the horizontal component of the earth's magnetism, we have from (16)

$$Hml \tan\delta=\Sigma(A)\frac{ml}{r^3}C$$

or

$$H \tan\delta=\frac{C\Sigma(A)}{r^3},$$

which gives

$$C=\frac{Hr^3}{\Sigma(A)}\tan\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Formula for ω.

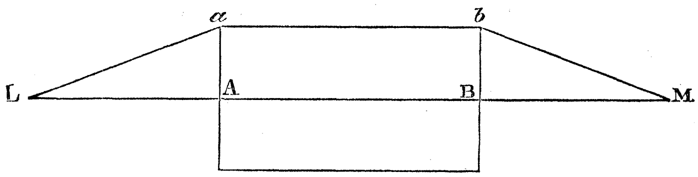
ω is the rotation expressed in circular measure corresponding to a difference of magnetic potential equal to unity. Thus

$$\omega=\frac{\theta}{V_L-V_M}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where θ is R expressed in circular measure, and V the magnetic potential at any part of the tube.

The following formula gives the difference of the magnetic potentials at the ends L, M, due to a current C.

If LM were infinite, it would be $4\pi n$C, where n is the number of windings. When the length of the tube is finite, and equal to LM,



we must subtract the following correction*:—

$$\frac{C\Sigma(A)}{\frac{1}{2}AB\,.\,Aa^2}\{La-LA-(Lb-LB)+Mb-MB-(Ma-MA)\}. \quad . \quad . \quad . \quad (21)$$

When LA is great compared with Aa,

$$La-LA=\tfrac{1}{2}\frac{Aa^2}{LA}-\tfrac{1}{8}\frac{Aa^4}{LA^3}+\&c.; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

or, retaining only the first term, we get

$$\frac{C\Sigma(A)}{AB}\cdot\left(\frac{1}{LA}-\frac{1}{LB}+\frac{1}{MB}-\frac{1}{MA}\right)=\frac{C\Sigma(A)}{AB}\left(\frac{AB}{LA\,.\,LB}+\frac{AB}{MA\,.\,MB}\right)$$

$$=C\Sigma(A)\left(\frac{1}{LA\,.\,LB}+\frac{1}{MA\,.\,MB}\right). \quad . \quad . \quad . \quad (23)$$

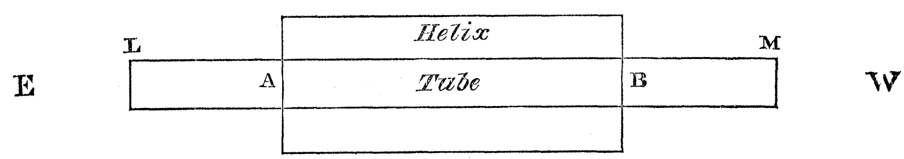
The corrected value is then

$$(V_L-V_M)=\left\{4\pi n-(\Sigma(A))\left(\frac{1}{LA\,.\,LA}+\frac{1}{MA\,.\,MB}\right)\right\}C \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Thus the complete formula for ω is

$$\omega=\frac{\theta}{\left\{4\pi n-(\Sigma(A))\left(\frac{1}{LA\,.\,LB}+\frac{1}{MA\,.\,MB}\right)\right\}\frac{Hr^3\tan\delta}{\Sigma(A)}}. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The following quantities were taken by measurement:—



AB=26·34 centims.
LA=18·95 „
BM=23·72 „

Distance from O *to centre of helix*=141·64 centims.; whence

$$OA=OB=r=\left\{\overline{141\cdot64}^2+\overline{13\cdot17}^2\right\}^{\frac{1}{2}},$$

or

$$r^3=2878499, \text{ and } \log r^3=6\cdot4591660.$$

* This formula is due to Professor MAXWELL.

Tan δ.

The scale, which consisted of a wooden board on which was pasted a printed paper scale of millims., was fixed on a massive wooden tripod-stand fixed to the wooden floor. The telescope was, as before mentioned, bedded with plaster of Paris in a block of stone cemented on to a brick pier. [N.B. All the brick piers and tables were built on the solid ground and came up through holes in the wooden floor.] The scale was so adjusted that, when no current was passing, the line from O to the figure seen in the telescope was perpendicular to the scale. This adjustment was made by hanging a plumb-line near the scale, between it and the mirror O, so that on looking into the telescope it was seen in the centre of the field. A second was then hung in the line between the first and the mirror. The edge of a **T** square being placed so as just to touch both lines, its base gave the direction of the scale.

As it was not certain whether or not the paper scale had stretched in the pasting, the deflections were determined by taking, with a pair of compasses, the distance between the zero reading and that corresponding to any current, and measuring it on the standard brass scale. The numbers thus obtained are hereafter spoken of as "corrected deflections."

The readings were taken in each direction. Their mean was called x.

The distance from O to the mean zero-point was called a, and by measurement we have

$$a=102 \cdot 50 \text{ centims.}$$

The values of $y=\tan\delta$, where δ is the deflection of the suspended magnet, were obtained by the following formula, due to Professor MAXWELL. We have

$$\begin{aligned} y=\tan\delta &= \tan\left(\tfrac{1}{2}\tan^{-1}\frac{x}{a}\right) \\ &= \sqrt{1+\frac{a^2}{x^2}}-\frac{a}{x} \\ &= \tfrac{1}{2}\frac{x}{a}-\tfrac{1}{8}\frac{x^3}{a^3}-\&\text{c.}^* \quad \ldots \ldots \ldots \ldots \quad (26) \end{aligned}$$

Determination of the Meridian.

The best way to have determined the meridian of the helix would have been to have deduced it from that of the suspended mirror. Owing, however, to the arrangement of the supports, this meridian could not be transferred to other parts of the table. The meridian was therefore determined by means of a needle furnished with a straw pointer 20 centims. long (by ELLIOTT), belonging to a large tangent galvanometer. Part of a

* This is a very simple formula to use, as we have

$$y=\tfrac{1}{2}\log^{-1}(\log x-\log a)-\tfrac{1}{8}\log^{-1}3(\log x-\log a).$$

sewing-needle was set upright in a hole drilled in the table, and filled with soft clay, at such a height that the magnetic needle when balanced on it just swung clear.

The latter was allowed to come to rest under a bell-glass, and then clamped by a simple arrangement. As the ends of the straw were 5 or 6 millims. from the table, a candle was placed close to the centre of the needle, and the ends of the shadow of the straw marked with a pencil. This meridian was then transferred to any other part of the table by means of parallel rulers*.

The following method was used for placing the helix at a known distance from the suspended magnet and perpendicular to the magnetic meridian, and for making that meridian which passed through the suspended magnet bisect the axis of the helix :—

A pencil line longer than the helix was drawn through the point on the meridian selected for the centre of the helix and perpendicular to that meridian. The plumb-bobs were hung at points, one near each end of this line and in it, and two more in the meridian passing through the suspended magnet.

A mark was made at the centre on the outside of the helix, and cross-wires placed at its ends inside. When the threads of the E.W. plumb-bobs sighted the cross-wires and the meridian-threads the point on its outside, the helix was properly placed.

Verification of the Meridian.

In order to see that there had been no error, the following verification was made, based on the principle that if the helix was placed symmetrically with regard to the meridian, the deflections produced in opposite directions by reversing the same current should be equal.

Readings.

Current direct.	No current.	Current reversed.
8·47	. . .	. . .
. . .	. . .	42·17
8·54	. . .	. . .
. . .	. . .	42·11
. . .	25·48	. . .

Corrected Deflections.

16·84	. . .	16·64

A difference too small to be worth correcting†.

The Light.

It was absolutely necessary that the light should be monochromatic, as the rotation of different colours for the same difference of magnetic potential is very different, varying

* See verification of meridian below.

† Professor MAXWELL has since pointed out to me that a slight displacement of the helix would not make so large a difference in the deflections as I had expected. I, however, leave in this paragraph for what it is worth.

nearly as λ^{-2}. A sodium flame was first tried but found not to be sufficiently steady; so it was decided to use a white light and a bisulphide-of-carbon prism. A very powerful paraffin-lamp with a duplex burner was finally chosen as the source of light. It was used with a lens and a slit, and threw a brilliant spectrum about 8 centims. long on a card. In the card was a vertical slit which admitted the light into the Nicol. By moving the prism, or lens, the spectrum could be moved along the card, and any desired colour admitted through the slit into the Nicol. In order to localize the light the lamp was removed from the lantern, and a spirit-lamp put in its place. The flame was coloured with thallium, and the green line projected on the card so as to fall upon the slit in it at minimum deviation. The lantern, lens, and prism being then clamped, the lamp was replaced, and the green light which fell upon the slit was of the same wave-length as that giving the thallium line. It was intended to make also observations at the sodium and lithium lines, but it was found impossible to isolate the sodium light, while to isolate the lithium the slits had to be made so narrow that the intensity was not sufficient to work by.

In employing the Jellett analyzer, if the light used was not *perfectly* monochromatic, the effect of the magnetic force was to colour the two halves of the field different colours, rendering all comparison of intensities impossible. After a great number of preliminary observations (continued for about seven months), all of which were rejected as I found means to improve the methods used, the following sets were made.

In these sets no readings were rejected, except on one occasion, when, finding the first few readings did not agree well with one another, I rejected the whole of them, and after a short rest began them again from the beginning.

In all the observations the Nicol was so placed that the division line of the "Jellett" was about horizontal, so that if there were any difference in the light admitted at the right and left of the field it should affect both halves equally.

Thus the light on leaving the lamp passed through the apparatus in the following order:—slit, lens, prism, slit, Nicol, collimator, tube, Jellett and its mountings.

"Current direct" is that direction of current which causes the north-pointing end of the suspended magnet to move toward the east.

After each reading, the circle was slightly displaced at random.

Set 1. February 25.—11 to 11.30 P.M. Green light $\lambda=(5\cdot349)10^{-5}$ centims. 7 cells.

Plane of polarization.	Scale-readings.	Corrected deflections.
No current.	25·45	
Current direct. 266° 12′ 0″ 266 12 0 *266 17 0 266 16 0 Mean 266 14 15†	3·71	21·63 centims.
Current reversed. 281 35 0 281 36 0 281 44 0 281 46 0 Mean 281 40 15	46·82	21·42 centims.

$2R=15^\circ\ 26'\ 0''$

$a=102\cdot5$ centims.

$x=$ mean of deflections $=21\cdot525$ centims.

$\frac{2R}{y}=8917'\cdot5$

$y=\tan\delta=\frac{1}{2}\frac{x}{a}-\frac{1}{8}\frac{x^3}{a^3}=\cdot10384$

Set 2. February 25.—11.30 to 12 P.M. Same light. 5 cells.

Plane of polarization.	Scale-readings.	Corrected deflections.
No current.	25·58	
Current direct. 268° 6′ 0″ 268 5 0 268 11 0 268 6 0 Mean 267 7 0	8·58	16·97
Current reversed. 280 11 0 280 9 0 280 1 0 280 1 0 Mean 280 5 30	42·06	16·50

$2R=11^\circ\ 58'\ 30''$

$a=102\cdot5$ centims.

$x=16\cdot735$ centims.

$\frac{2R}{y}=8861'\cdot1$

$y=\cdot081090$

* With regard to the differences between different determinations of the plane of polarization, it must be remembered that it is impossible to keep the battery-current absolutely constant during a series of experiments extending over 10 or 12 minutes.

† The seconds only appear in the means.

Set 3. February 26.—0 to 0.30 A.M. Same light. 6 cells.

Plane of polarization.	Scale-readings.	Corrected deflections.
No current.	25·51	
Current direct. 267° 17′ 0″ 267 26 0 267 26 0 267 19 0 Mean 267 22 0	6·40	19·09 centims.
Current reversed. 280 57 0 281 5 0 280 59 0 281 5 0 Mean 281 1 30	44·00	18·48 centims.

$2R = 13° 39' 30''$ $a = 102·5$ centims.

$x = 18·785$ centims.

$\frac{2R}{y} = 8820'·6$ $y = ·092907$

Tabulating the results, we have for $\frac{2R}{y}$

Set.	2R	$\frac{2R}{y}$	Cells.
2.	11° 58′ 30″	8861·1 minutes.	5
3.	13 39 30	8820·6 „	6
1.	15 26 0	8917·5 „	7

See p. 32.

To reduce the result to the D line, we may use one of the formulæ given by M. VERDET*; but as they are only approximate, I have thought it best to state the final result for the thallium line, wave-length

$$(5·349)10^{-5}\dagger,$$

on which the experiments were made. The wave length of the D line is

$$(5·892)10^{-5}\dagger.$$

Value of H.

The ratio of H at Dorking to H at Kew was determined by vibrating the same magnet at the two places, and determining the times of vibration. We then know that the values are inversely as the squares of the times of vibration.

Mr. WHIPPLE, Director of the Magnetic Observatory at Kew, had the great kindness,

* Œuvres, tom. i. p. 211. † WATTS' Index of Spectra.

not only to lend me a magnet, but also to have it vibrated at Kew, and to determine the temperature coefficient and the correction for torsion of the thread.

The magnet was, at Pixholme, suspended to the same stone support and observed with the same telescope and scale as the smaller magnet used to determine the strength of the currents. It was therefore in exactly the same place as that magnet had been.

The method used for taking the time of vibration was explained to the author by Prof. MAXWELL, in a letter from which the following paragraphs are extracted*:—

"The best way to take the times is to begin with getting the approximate time of an odd number of half-vibrations 1, 3, or 5, according to convenience. Take such an odd number as will have a period of nearly half a minute or 20″, or something easily reckoned, then note down the time of passage through a point near the point of rest, marking it + when it goes to right, and − when it goes to left, and observe, say, 11 passages, 6+ and 5−, at convenient intervals. Then after a time equal to that of 100 vibrations or near it, begin again and observe 11 passages.

"From the two pairs of 11 passages find the time of vibration roughly, and thence the number of passages between the first and second set. Finally, find the time by dividing the mean interval between the first and second set by the number of passages. By taking the passages alternately + and −, you eliminate the error arising from taking the passage at a wrong point of the swing."

A large clock beating seconds was so placed that the observer at the telescope could see it by merely looking round. The times of passage were taken as follows:—

The hour and minute having been written down by an assistant, the observer noted the second, and went on counting the beats by ear. Then looking into the telescope, he observed the time of every 7th passage, and called it out to be written down by the assistant. The passages were thus always alternately + and −. The time occupied by 7 passages was sufficient to allow of the clock being looked at to get a fresh beat between each.

The approximate times at which each passage might be expected, within, say, 3″, were previously written out and placed in a convenient position. This prevented the possibility of a 5th or 9th passage being observed instead of a 7th. A thermometer hung in the case (wood with glass front) in which the magnet swung gave the temperature.

The following Tables give the results of the experiments—that of "Vibrations at Kew" being sent by Mr. WHIPPLE, and those of "Vibrations at Pixholme" being determined by the author.

* See also MAXWELL'S 'Electricity,' Art. 456, vol. ii. p. 102.

Kew Observatory.

Observations of Vibration of Magnet $n\psi$ lent to Mr. J. E. H. GORDON.

Date.	Number of Observations.	Time of commencement.	Mean time of one vibration.	Torsion of thread $1+\frac{\tau}{H_m}$ *.	Mean Temperature.
1876.		h m s	sec.		°
April 11......	1.	2 43 33	3·6880	1·00061	51·4
" 11......	2.	3 28 41	3·6868	1·00067	52·8
" 11......	3.	4 1 1	3·6866	1·00067	52·7
" 13......	4.	11 6 29	3·6850	1·00069	40·3
" 13......	5.	11 34 26	3·6858	1·00069	42·4
" 13......	6.	11 49 37	3·6859	1·00069	43·2

$q=0·000178$.

$q'=0·000000596$.

q_0=the correction for the decrease of the magnetic moment of the magnet produced by an increase of temperature, and is expressed by a formula of the form—

$$\text{correction to } t_0=q(t_0-t)+q'(t_0-t)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

t_0 being the observed temperature, and t an adopted standard temperature.

Vibration Experiments at Pixholme.

From some preliminary experiments which need not be detailed it was found that the time of a half-vibration at Pixholme was approximately 3·693 seconds.

The following Table gives the time of every 7th passage taking 6+ and 5−.

Set 1 (A).

April 23, 1876, P.M. Mean temperature 13°·9 C.

	Passages in + direction.	In − direction.
	h m s	h m s
	6 22 3½	6 22 29
	6 22 55	6 30 20½
	6 23 46½	6 24 12½
	6 24 38¼	6 25 4
	6 25 30	6 25 56
	6 26 21½	
Mean . .	6 24 12·458	6 24 12·400

Time of middle negative passage:—

	h m s
From + observations	6 24 12·458
From − "	6 24 12·400
Mean	6 24 12·429

* I have here altered the Kew notation to suit the letters commonly used in the C.G.S. system.

Set 1 (B).

	Passages in + direction.	In − direction.
	h m s	h m s
	6 39 2	6 39 28
	6 39 53½	6 40 19½
	6 40 45¼	6 41 11⅓
	6 41 37	6 42 3
	6 42 28½	6 42 54½
	6 43 20¼	
Mean . .	6 41 11·83	6 41 11·26

Time of middle negative passage:—

	h m s
From + observations	6 41 11·83
From − „	6 41 11·26
Mean	6 41 11·545

Interval between mean negative passages A and B, 16 min. 59·251 sec.

If we divide by 3·693 the approximate period of a half-vibration, the nearest even whole number to the quotient will be the number of passages in the time.

Dividing by 3·693 we obtain the quotient 275·99, which gives 276 half-vibrations.

Dividing 1019·25 sec. by 276 we obtain for the time of one half-vibration 3·6929 seconds.

Set 2 (A).

April 25, A.M. Mean temperature 11°·9 C.

	Passages in + direction.	In − direction.
	h m s	h m s
	9 53 3	9 53 28⅓
	9 53 54¼	9 54 20
	9 54 46¼	9 55 12
	9 55 38	9 56 4
	9 56 29½	9 56 55½
	9 57 21¼	
Mean . .	9 55 12·0416	9 55 11·9666

Time of middle negative passage:—

	h m s
From + observations . .	9 55 12·0416
From − „ . .	9 55 11·9666
Mean . . .	9 55 12·0042

Set 2 (B).

Passages in + direction.	In − direction.
h m s	h m s
10 9 $3\frac{1}{2}$	10 9 $29\frac{1}{3}$
10 9 $55\frac{1}{4}$	10 10 21
10 10 47	10 11 13
10 11 $38\frac{1}{2}$	10 12 $4\frac{1}{2}$
10 12 $30\frac{1}{4}$	10 12 56
10 13 22	
Mean . . 10 11 12·750	10 11 12·766

Time of middle negative passage:—

	h m s
From + observations . .	10 11 12·750
From − „ . .	10 11 12·766
Mean	10 11 12·7833

Interval between mean negative passages A and B, 16 min. 0·7791 sec.

Number of passages =260·1, that is 260.
Time=3·6953 sec.

Set 3 (A).

April 28, A.M. Mean temperature 11°·5 C.

Passages in + direction.	In − direction.
h m s	h m s
10 34 0	10 34 $25\frac{1}{2}$
10 34 $51\frac{1}{2}$	10 35 $17\frac{1}{4}$
10 35 $43\frac{1}{3}$	10 36 $9\frac{1}{4}$
10 36 35	10 37 1
10 37 27	[10 37 $52\frac{3}{4}$]*
10 38 $18\frac{1}{2}$	
Mean. . 10 36 9·222	10 36 9·150

Time of middle negative passage:—

	h m s
From + observations	10 36 9·222
From − „	10 36 9·150
Mean	10 36 9·186

* Interpolated.

Set 3 (B).

	Passages in + direction.	In — direction.
	h m s	h m s
	10 48 2	10 48 28
	10 48 55	10 49 20
	10 49 46	10 50 $11\frac{1}{2}$
	10 50 $37\frac{1}{2}$	10 51 $3\frac{1}{4}$
	10 51 29	10 51 55
	10 52 21	
Mean . .	10 50 11·750	10 50 11·550

Time of middle negative passage:—

	h m s
From + observations	10 50 11·750
From — ,, ...	10 50 11·550
Mean	10 50 11·650

Interval between mean negative passages A and B, 14 min. 2·564 sec.

Number of passages	228·1=228.
Time	3·6954 sec.

Set 4 (A).

April 28, P.M. Mean temperature 12°·9 C.

	Passages in + direction.	In — direction.
	h m s	h m s
	2 1 5	2 1 $30\frac{1}{2}$
	2 1 $56\frac{1}{3}$	2 2 22
	2 2 48	2 3 14
	2 3 40	2 4 $5\frac{1}{2}$
	2 4 $31\frac{1}{2}$	2 4 $57\frac{1}{4}$
	2 5 23	
Mean . . .	2 3 13·972	2 3 13·850

Time of middle negative passage:—

	h m s
From + observations	2 3 13·972
From — ,, ...	2 3 13·850
Mean	2 3 13·911

Set 4 (B).

	Passages in + direction.	In − direction.
	h m s	h m s
	2 12 2	2 13 28
	2 12 54	2 13 19½
	2 13 45½	2 14 11⅓
	2 14 37	2 15 3
	2 15 29	2 15 55
	2 16 20½	
Mean . .	2 14 11·333	2 14 11·366

Time of middle negative passage:—

	h m s
From + observations	2 14 11·333
From − „	2 14 11·366
Mean	2 14 11·350

Interval between mean negative passages A and B, 10 min. 57·439 sec.

Number of passages	178·0
Time	3·6935 sec.

Set 5 (A).

April 28, P.M. Mean temperature 13°·6 C.

	Passages in + direction.	In − direction.
	h m s	h m
	5 45 1¼	5 45 27
	5 45 53	5 46 19
	5 46 44½	5 47 10¼
	5 47 36¼	5 48 2
	5 48 28	5 48 54
	5 49 20	
Mean . .	5 47 10·500	5 47 10·450

Time of middle negative passage:—

	h m s
From + observations	5 47 10·500
From − „	5 47 10·450
Mean	5 47 10·4750

Set 5 (B).

	Passages in + direction.	In − direction.
	h m s	h m s
	5 55 59	5 56 $24\frac{1}{4}$
	5 56 $50\frac{1}{4}$	5 57 $16\frac{1}{4}$
	5 57 42	5 58 8
	5 58 34	5 59 0
	5 59 $25\frac{1}{2}$	5 59 $51\frac{1}{4}$
	6 0 $17\frac{1}{4}$	
Mean . .	5 58 8·00	5 58 7·97

Time of middle negative passage:—

	h m s
From + observations	5 58 8·00
From − ,, ...	5 58 7·97
Mean	5 58 7·985

Interval between mean negative passages A and B, 10 min. 57·510 sec.

Number of passages 178
Time . . . . 3·6938 sec.

CORRECTIONS TO VIBRATION TIME.

Temperature Correction.

The Kew formula was used, and as the coefficients of the magnet had been determined at Kew for the Fahr. scale, that scale was used in the work.

The square of the time of vibration varies inversely as the magnetic moment. Hence if the temperature of the magnet be greater than the standard temperature, the observed value of the vibration time will be too great, and its square must be divided by $(1+q_0)$, where q_0 is the correction given by the Kew formula.

The temperatures at which the vibration observations were taken were:—

At Kew.

No. Obs.	Temp. F.
K 1.	51°·4
2.	52·8
3.	52·7
4.	40·3
5.	42·4
6.	43·2

At Pixholme.

No. Obs.	Temp. F.
P 1.	57°·0
2.	53·4
3.	52·7
4.	55·2
5.	56·5

Let t the adopted standard temperature be that of K (4), which, being the lowest in either set, will make all the terms positive.

The following are then the values of q_0 :—

$$K(1),\ q_0 = \cdot000178(51^\circ\cdot4 - 40^\circ\cdot3) + \cdot000000596(51^\circ\cdot4 - 40^\circ\cdot3)^2 = \cdot0020492.$$

Similarly :—

K (2) . .	q_0 = ·0023181	P (1) . .	q_0 = ·0029908
K (3) . .	„ = ·0022988	P (2) . .	„ = ·0023420
K (4) . .	„ = 0 [standard]	P (3) . .	„ = ·0022988
K (5) . .	„ = ·0003586	P (4) . .	„ = ·0026654
K (6) . .	„ = ·00051895	P (5) . .	„ = ·0028942

Torsion of the Suspending Thread.

We require the ratio of the force of torsion to the magnetic directive force.

The Kew formula is (changing the notation to suit the letters commonly used in the C.G.S. system)

$$\frac{\tau}{H_m} = \frac{u^\circ}{90^\circ - u^\circ},$$

where τ is the force of torsion and u° is the change of declination produced by a twist of 90° in the torsion-thread.

As the magnet was not suspended from a torsion-circle, the only twist that could be applied was $\pm n$ (360°) given by twisting the magnet completely round in a $\mp$ direction.

If, however, which for so small a correction as this is we may do, we assume that the change of declination produced by a twist of 360° is 4 times that produced by 90°, the formula still holds, only we must substitute 360° for 90° in the denominator; we have then

$$\frac{\tau}{H_m} = \frac{u^\circ}{360^\circ - u^\circ}, \quad \ldots \ldots \ldots \ldots \quad (28)$$

where u° is now the torsion produced by 360° of twist.

The following observations were made, the first by reading the scale when the magnet was at rest, the others by taking the limits of swing after stopping the magnet as nearly as possible by means of a magnetized penknife :—

Obs.		Scale-readings in centims.				Mean.
1.	No torsion	16·23				16·23
2.	Twist of 360° + ...	13·28 15·88	13·36 15·85	13·40 15·81	13·40 15·80	
		14·580	14·605	14·605	14·600	14·5975
3.	No torsion, viz. since (2) magnet turned 360° (–)	14·70 17·90	14·70 17·88	14·75 17·80	14·75 17·80	
		16·30	16·29	16·275	16·275	16·2850
4.	Twist of 360° (–).........	15·48 20·08	15·50 20·04	15·52 20·00	15·53 20·00	
		17·780	17·770	17·776	17·765	17·7687
5.	No torsion, viz. since (4) magnet turned 360° +	13·70 19·20	13·82 19·10	13·90 19·10	13·92 19·04	
		16·49	16·46	16·50	16·48	16·4825

Mean zero from (1) and (3)=16·2575. From (2) this gives that 360° of twist + causes a deflection corresponding to 1·660 centim. on the scale.

Mean zero from (3) and (5)=16·3837, which from (4) gives a corresponding deflection of 1·385 centim.

Mean deflection therefore =1·5225 centim.

To obtain the number of minutes corresponding to this, we have the formula of (26), p. 16,

$$\tan u = \tfrac{1}{2}\frac{x}{a} - \tfrac{1}{8}\frac{x^3}{a^3},$$

where $a=102{\cdot}5$ centims., and $x=1{\cdot}5225$ centim.

This gives $\tan u = {\cdot}0074251,$

which from the tables gives the angle in minutes by which the declination is changed by a twist of 360°, viz.

$$u=25'{\cdot}46\,;$$

taking $\frac{1}{4}$ of this for the torsion produced by 90° of twist we have, from the Kew tables,

$$u=6'{\cdot}365 \text{ corresponds to } 1+\frac{\tau}{\mathrm{H}_m}=1{\cdot}00118.$$

Clock Rate.

The Kew observations are corrected for clock rate.

Owing to an accident to the comparing chronometer, the rate of the Pixholme clock was not determined till two or three days after the conclusion of the vibration experiments.

The Kew formula of correction is

$$\text{Real time of vibration} = (\text{observed time})\left(1-\frac{s}{86400}\right) \quad . \quad . \quad . \quad . \quad (29)$$

where s is the daily rate in seconds, being $+$ when clock gains, $(-)$ when it loses.

On April 29, at 4 P.M. the clock was 7 minutes 5 seconds slow*; on May 3, at 4 P.M. the clock was 9 minutes 30 seconds slow. Giving a rate of

$$s = -36 \cdot 25\,;$$

viz. it lost $36\frac{1}{4}$ seconds per day.

By interpolation from the Kew Tables we have

$$\left(1-\frac{s}{86400}\right) = 1 \cdot 0004225$$

To obtain the times of vibration at 40°·3 we have

$$\text{Log (vibration time at } 40^\circ{\cdot}3)^2 = 2 \log (\text{observed time}) - \log (1+q_0). \quad . \quad (30)$$

Also the torsion causes an apparent increase of magnetic force, and therefore decreases the time of vibration, whence

$$\text{Log (vibration time corrected for torsion)}^2 = 2 \log (\text{observed time}) - \log \left(1+\frac{\tau}{H_m}\right). \quad (31)$$

Also

$$\text{Log (vibration time corrected for clock error)} = \log (\text{observed time}) + \log \left(1+\frac{s}{86400}\right). \quad (32)$$

Combining these three formulæ, we have for the true time of vibration corrected for temperature, torsion, and clock—

$$\text{Log (true vibration time)}^2 = 2\left(\log (\text{observed time}) + \log \left(1-\frac{s}{86400}\right)\right)$$
$$-\log (1+q_0) - \log \left(1+\frac{\tau}{H_m}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

We have the following values for the corrected vibration times:—

At Kew.

Set.	Log of observed time of vibration.	Log $(1+q_0)$.	Log $\left(1+\frac{\tau}{H_m}\right)$.	Log of square of true time of vibration.
1.	0·5667909	0·0008894	0·0002647	1·1324277
2.	0·5666496	0·0010057	0·0002909	1·1320026
3.	0·5666260	0·0009971	0·0002909	1·1319649
4.	0·5664375	0·0000000	0·0002996	1·1325754
5.	0·5665318	0·0001557	0·0002996	1·1326083
6.	0·5665435	0·0002253	0·0002996	1·1325621

* These observations were taken by the Dorking clock-maker.

At Pixholme.

Set.	$\text{Log}\left(1+\frac{\tau}{H_m}\right)=0{\cdot}0005122.$ $\text{Log}\left(1-\frac{s}{86400}\right)=0{\cdot}0001835.$		
	Log of observed time of vibration.	Log $(1+q_0)$.	Log of square of true time of vibration.
1.	0·5673675	0·0003003	1·1342895
2.	0·5676497	0·0010159	1·1341383
3.	0·5676615	0·0009971	1·1341807
4.	0·5674381	0·0001559	1·1345751
5.	0·5674734	0·0002550	1·1345466

From these data we get the ratio of the magnetic force at Pixholme to that at Kew, for

$$\frac{\text{H at Pixholme}}{\text{H at Kew}}=\frac{(\text{H at Kew at time of Kew vibrations})\ .\ (\text{Vibration time at Kew})^2}{(\text{H at Kew at time of Pixholme vibrations})\ .\ (\text{Vibration time at Pixholme})^2} \quad (34)$$

and the values of H at Pixholme at the times of the optical experiments equal the values at Kew at those times multiplied by the above ratio.

The last columns of the two following Tables supply respectively the numerator and denominator of the above fraction.

Set.	Dates of vibration experiments at Kew.	Mean value of H at Kew	(Corrected square of vibration time at Kew) × (Mean value of H at Kew)
		at these dates.	
1.	1876. April 11, 3 P.M.	0·179109	2·42964*
2.	,, ,, 3.30 P.M.	0·179132	2·42759
3.	,, ,, 4.0 P.M.	0·179146	2·42758
4.	April 13, 11 A.M.	0·178898	2·42768
5.	,, ,, 11.30 A.M.	0·178856	2·42618
6.	,, ,, Noon	0·178866	2·42711
		Mean	2·42723

* Rejected.

Set.	Dates of vibration experiments at Pixholme.	Mean value of H at Kew	(Corrected square of vibration time at Pixholme) × (Mean value of H at Kew)
		at these dates.	
1.	1876. April 23, 6.30 P.M.	0·179368	2·44362
2.	April 25, 10 A.M.	0·178935	2·43688*
3.	April 28, 10.30 to 10.50 A.M.	0·179024	2·44395
4.	,, ,, 2.2 to 2.15 P.M. ...	0·179190	2·44281
5.	,, ,, 5.45 to 6 P.M.	0·179245	2·44339
		Mean	2·44344

This gives

$$\text{H at Pixholme} = (\cdot 993366)\ \text{H at Kew}.$$

But at the dates of the optical experiments H had, at Kew, the values given in the second column of the following Table, and therefore at Pixholme those in the third.

Set.	Dates of optical experiments at Pixholme, 1876.	Mean values of H at Kew	Mean values of H at Pixholme
		at these dates.	
1.	Feb. 25, 11 to 11.30 P.M.	0·179117	0·177929
2.	Feb. 25, 11.30 P.M. to Midnight	0·179241	0·178051
3.	Feb. 25, Midnight, to Feb. 26, 0.30 A.M. ...	0·179244	0·178055

Dividing the three values of $\frac{2R}{y}$ given on page 20 by these numbers respectively, we have, for the three quantities which ought to be identical,

$$\frac{2R}{yH} = (1)\ 50118\cdot 4$$

$$(2)\ 49767\cdot 0$$

$$(3)\ 49538\cdot 7$$

$$\text{Mean} \ldots \ 49808\cdot 0$$

Extreme difference from the mean 0·6 per cent.

We have, finally, then, if ω be the rotation in bisulphide of carbon of the plane of polarization of the ray whose wave-length is

$$(5\cdot 349)10^{-5}$$

* Rejected.

between two points whose magnetic potentials differ by unity,

$$\omega=\frac{\frac{k2R}{\tan\delta}\cdot\frac{1}{H}\cdot\Sigma(A)}{r^3\left\{4\pi n-\Sigma(A)\left(\frac{1}{LA\,.\,LB}+\frac{1}{MA\,.\,MB}\right)\right\}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where k is the number of units of circular measure in a minute of arc $=\frac{3{\cdot}14159}{180\times 60}$;

or $$\omega=\frac{(\cdot 00029088)(77453)}{2878499\left\{4\pi(1028{\cdot}15)-77453\left(\frac{1}{(18{\cdot}95)(45{\cdot}29)}+\frac{1}{(50{\cdot}06)(23{\cdot}72)}\right)\right\}}\cdot\frac{2R}{\tan\delta\,.\,H}$$

$$=(\log^{-1}\overline{10}{\cdot}7866630)(49808{\cdot}0)$$

$$=\log^{-1}\overline{5}{\cdot}4839621$$

$$=3{\cdot}04763(10^{-5}).$$

As the constant is the ratio of a number to a current, its dimensions are

$$[\omega]=[M^{-\frac{1}{2}}L^{-\frac{1}{2}}T].$$

This is VERDET'S constant in absolute measure. For light of a given wave-length passing through a given substance, it is a fixed and definite physical quantity, depending only on the units of length, mass, and time*.

It is the number in optical measure which is equivalent to unity in electro-magnetic measure. In future investigations quantities expressed in electro-magnetic measure can be expressed in optical measure by multiplying them by this number.

ω, then, is defined to be VERDET'S constant for the thallium ray in bisulphide of carbon, expressed in C.G.S. measure.

I here insert, as I have been requested to do, the result of my former paper on the same subject. It is that for distilled water with white light,

$$\omega=4{\cdot}496(10^{-6})\dagger.$$

I do not, however, attach much value to the result, as the different determinations are in the ratios of

7·563, 7·406, 8·295, 8·401, 6·916,

showing variations from the mean of ± 7 per cent., or giving only about $\frac{1}{10}$ of the accuracy of the present paper.

The methods used for determining the constants were also susceptible of less accuracy.

Before concluding this paper, I must express my thanks, first and chiefly, to Professor MAXWELL, who has superintended every detail of the work for the year and eight months

* The magnetic rotative power of bisulphide of carbon here comes in the same way as the specific heat of water comes into JOULE'S equivalent.

† By an error in arithmetic this was printed 10^{-7} in the abstract of my paper published Proc. Roy. Soc. June 1875.

during which it has been in progress; and secondly, to Mr. WHIPPLE, Director of the Kew Magnetic Observatory, who sent me all the data for calculating the value of H, and had several series of experiments and calculations made for me at the Observatory.

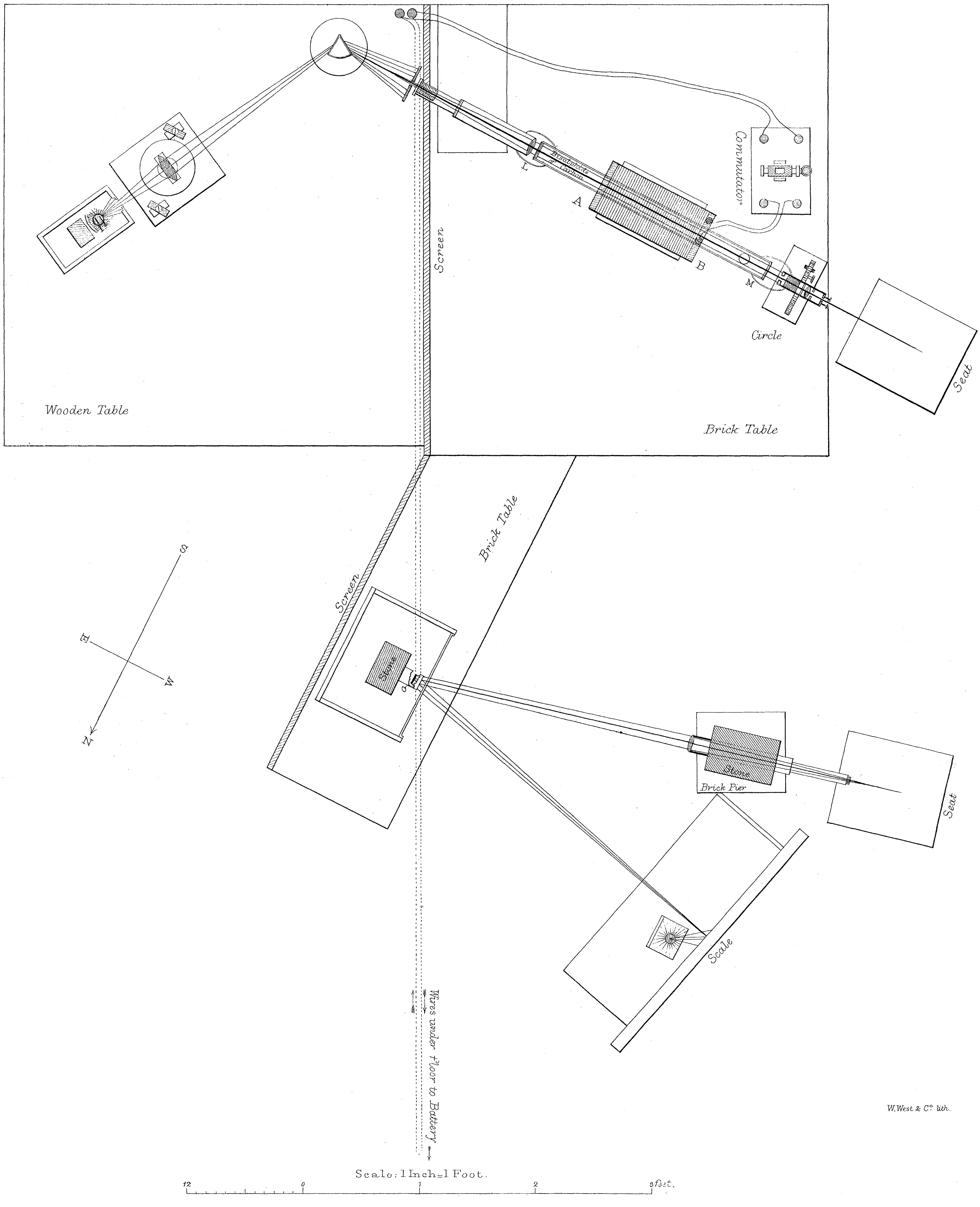
APPENDIX.

Analysis of the Bisulphide of Carbon employed.—Mr. J. M. THOMSON, Demonstrator of Chemistry at King's College, London, had the kindness to test the purity of the bisulphide for me. He writes:—"As far as I can see, the CS_2 you sent me is perfectly good. Its sp. gr. (the mean of three determinations) is 1·275 at 15°, the sp. gr. of pure CS_2 being 1·271 at 15°. Its boiling point is rather above 47°, so that the sample may be considered as very nearly pure. It leaves a trace of sulphur on evaporation, but not a quantity that can be weighed."

During the year which elapsed between the experiments and the testing of the bisulphide the latter was kept in a stoppered bottle of thick blue glass.

The temperature of the bisulphide, at the time of the optical experiments, was unfortunately not taken; but I believe it was about 55° F., as it was a frosty night, while a good fire kept the laboratory pleasantly warm.

The logarithms used in the calculations are from CHAMBERS's Mathematical Tables, Ed. 1873.


Wooden Table
Screen
Bisulphide of Carbon
L
A
B
M
Commutator
Circle
Seat
Brick Table
Brick Table
Screen
Stone
a
Brick Pier
Stone
Seat
Scale
S
E
W
N
Wires under Floor to Battery
Scale: 1 Inch = 1 Foot.
12
0
1
2
3 feet.

W. West & C°. lith.